e-content for students

B. Sc.(honours) Part 2 paper 4

Subject:Mathematics

Topic:Reduction of a force system to a force & couple

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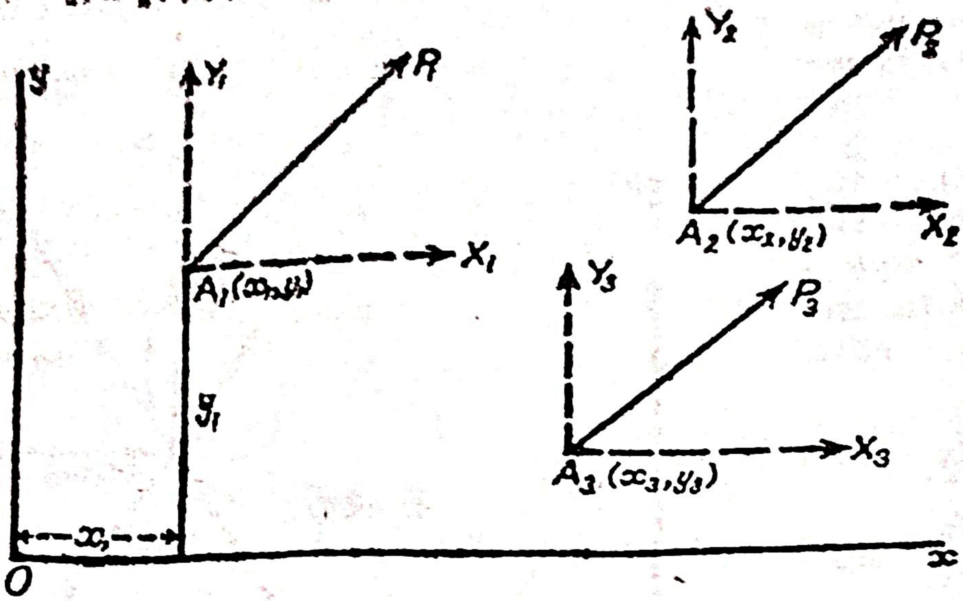
Theorem:prove that any system of Coplanar forces acting on a rigid body is equivalent to a single force acting at an arbitrary point in the plane of the forces together with couple

Let P_1, P_2, P_3......be the given system of forces acting at the



points A_1, A_2, A_3......respectively, the co-ordinates of A_1, A_2, A_3, ... being (x_1, y_1), (x_2, y_2), (x_3, y_3)...... with respect to the fixed axes Ox and Oy in the plane of the forces.

First resolve P_1 into components X_1 and Y_1 parallel to Ox and Oy.

Now Y_1 can be replaced by

Y_1 at O and a couple Y_1x_1.

X_1 can be replaced by

X_1 at O and a couple $-X_1y_1$.

Hence P_1 is replaced by

X_1 along Ox, Y_1 along Oy,

and a couple $Y_1x_1 - X_1y_1$.

Hence, adding up all the forces, we have

ΣX_1 along Ox,

ΣY_1 along Oy,

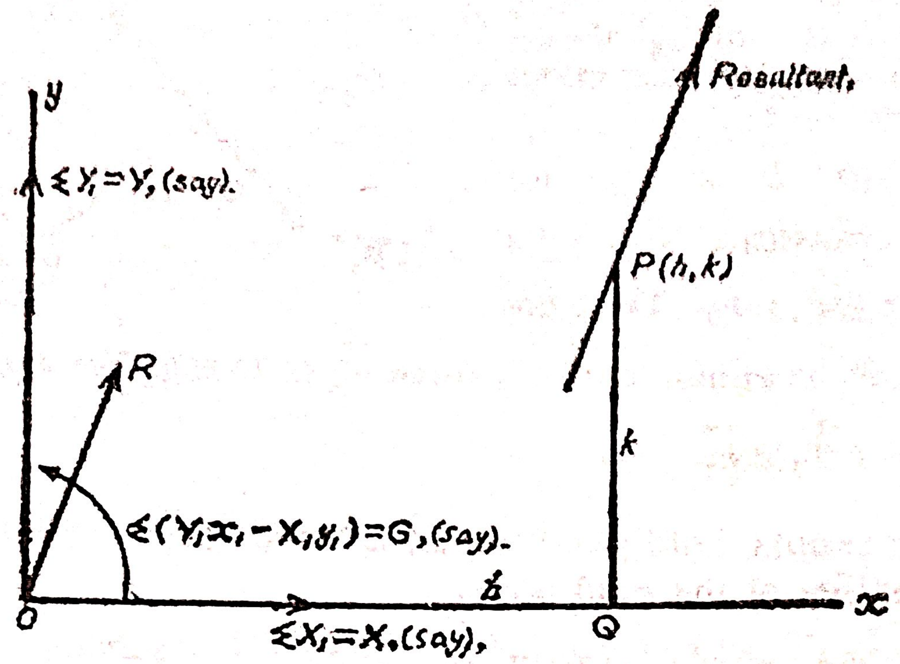
and a couple $\Sigma(Y_1x_1 - X_1y_1) = G$, (say).

ΣX_1 and ΣY_1 are further equivalent to a single force R acting at O.

Theorem

Obtain the equation to the line of a ction of the resultant of system of coplanar forces

We know that a system of coplanar forces acting on a rigid body can be reduced to a single force, R, acting at an arbitrary chosen point, O, in the plane of the forces together with a couple, G.



Let P be any point (h, k) which lies on the resultant of the given system.

The moment of the system about P

$$= \text{the moment of the resultant about } P = 0,$$

i.e. $\quad G + X.PQ - Y.OQ = 0$; i.e. $G + X.k - Y.h = 0$.

Hence the locus of (h, k) is

$$\boxed{\mathbf{G + Xy - Yx = 0,}}$$

which is the required equation of the line of action of the resultant force.

Theorem

Obtain the general conditions of equilibrium of system of forces acting in one plane upon a rigid body

If the system of forces be in equilibrium, then

$$R=0 \text{ i.e. } \sqrt{X^2+Y^2}=0 \text{ i.e. } X^2+Y^2=0$$

which gives $X=0$ and $Y=0$, and also $G=0$.

Conversely, If $X=0,\ Y=0$ and $G=0$

then $R=0$ and $G=0$.

Thus the system of forces is in equilibrium. Hence the necessary and sufficient conditions for equilibrium are

$$\boxed{\mathbf{X=0,\ Y=0 \text{ and } G=0.}}$$